

# Mass Screening in Modified Gravity

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## Abstract

Models of modified gravity introduce extra degrees of freedom, which for consistency with the data, should be suppressed at observable scales. In the models that share properties of massive gravity such a suppression is due to nonlinear interactions: An isolated massive astrophysical object creates a halo of a nonzero curvature around it, shielding its vicinity from the influence of the extra degrees of freedom. We emphasize that the very same halo leads to a screening of the gravitational mass of the object, as seen by an observer beyond the halo. We discuss the case when the screening could be very significant and may rule out, or render the models observationally interesting.

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# 1 Introduction and summary

One of the most puzzling discoveries of our times is the fact that the present-day expansion of the Universe is accelerating [1]. Such an acceleration can be attributed to the existence of “dark energy” - a substance with a negative enough pressure - that is present in the Universe and, hence, in the rhs of the Einstein equation:

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{dark energy}}), \quad (1)$$

where  $G_{\mu\nu}$  stands for the Einstein tensor of the 4D space-time with metric  $g_{\mu\nu}(x)$ , and  $T_{\mu\nu}^{\text{matter}}$  and  $T_{\mu\nu}^{\text{dark energy}}$  denote the stress-tensors for visible and dark matter, and dark energy, respectively.

On the other hand, one can consider a different logical possibility: that the accelerated expansion is due to modified General Relativity (GR)<sup>3</sup>. In this case, we would have the modified Einstein equations of the form:

$$G_{\mu\nu} - \mathcal{K}_{\mu\nu}(g, m_c) = 8\pi G_N T_{\mu\nu}^{\text{matter}}, \quad (2)$$

where  $\mathcal{K}_{\mu\nu}(g, m_c)$  denotes a tensor that could depend on a metric  $g$ , its derivatives, as well as on other fields not present in GR. Moreover,  $\mathcal{K}$  depends on a dimensionful constant  $m_c \sim H_0 \sim 10^{-42} \text{ GeV}$ , that sets the distance/time scale  $r_c \equiv m_c^{-1}$  at which the Newtonian potential significantly deviates from the conventional one. For instance, in the DGP model [4]  $\mathcal{K}_{\mu\nu}$  is related to the extrinsic curvature tensor that gives rise to a self-accelerated solution [5, 6] (see, the comments on viability of this solution at the end of this section, and Ref. [7] for a recent review).

Even though the difference between (2) and (1) might seem just conventional at a first sight, in reality, however, it could be observationally significant. For instance, it is possible to choose the rhs of (1) so that it gives rise to a background evolution obtained from (2), nevertheless, perturbations on these backgrounds would be different in (2) and (1), see, e.g., [8]-[16].

In what follows we will focus on the issue of whether the two approaches, (2) and (1), could be differentiated by properties of a Schwarzschild-like solution for a static spherically symmetric source.

A key feature of any theory of modified gravity of the form (2) is that, unlike GR, it allows for the possibility of having non-vanishing curvature outside of a source. This can be easily understood by taking the trace of (2) in a region outside of sources (where  $T_{\mu\nu} = 0$ ), that gives:

$$-R = \mathcal{K}, \quad (3)$$

where  $\mathcal{K} = \mathcal{K}^\mu_\mu$  needs not be zero. This fact affects in particular the notion of mass and leads to the interesting phenomenon of *screening*. For example, when defining

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<sup>3</sup>The latter approach is motivated by the “old cosmological constant problem” (for a review of which, see, e.g., [2], and in the context of modified gravity see, e.g., [3]). We will not be discussing this problem in the present work.

the Komar mass (which coincides with the ADM mass in stationary asymptotically flat space-time) one starts with an expression in terms of a volume integral of a projection of the Ricci tensor (see, e.g., [17]) and then by use of Einstein's equations the Ricci tensor is replaced by  $T_{\mu\nu} - g_{\mu\nu}T/2$ . In a modified gravity theory of the form (2) an extra term containing  $\mathcal{K}_{\mu\nu}$  and its trace is generated in the replacement of the Ricci tensor. Thus, the definition of the Komar mass contains an extra piece, referred to as a mass deficit below, which for a compact static source of spherical symmetry is given by

$$\Delta M \propto M_{Pl}^2 \int dr^3 \left( \mathcal{K}_{00} - \frac{1}{2} g_{00} \mathcal{K} \right) . \quad (4)$$

How significant is the mass deficit? The answer to this question would depend on a concrete model at hand. However, we would like to argue that in models which share properties of Lorentz invariant “massive gravity”, the mass deficit could be of the order of the mass itself. This has something to do with the fact that such models exhibit the so called strongly coupled behavior [18, 19] (see also [20, 21, 22, 23], and section 2 below for a summary), in spite of the fact that gravitational fields is weak everywhere [19, 24, 25, 26].

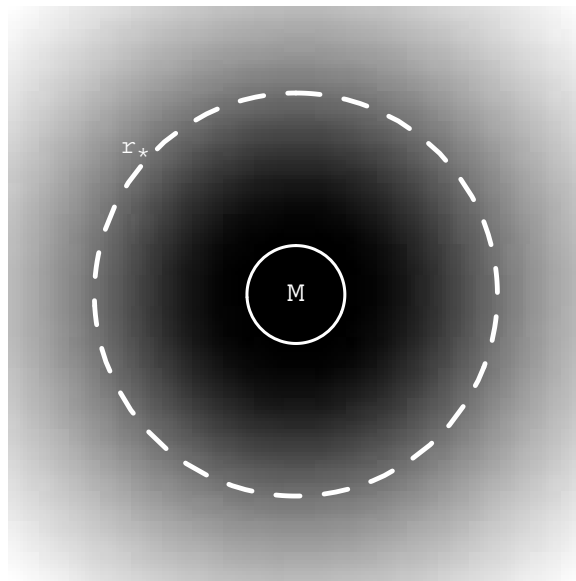


Figure 1: Curvature extends outside a source to a distance  $r_*$ .

One way to interpret this property for a static spherically symmetric source is to observe [27] that the source gives rise to the curvature that extends up to a macroscopic distance  $r_*$ , in the way depicted in figure 1. This curvature contributes to the integral (4).

In a concrete case of the non-perturbative Schwarzschild solution of the DGP model [27], we will show that this integral is saturated around a distance  $r_*$  ( $\sim$

$(M/m_c^2 M_{Pl}^2)^{1/3}$  - the topic of the next section ) where the value of  $\mathcal{K}$  is typically of order  $m_c^2$ . Then, the result of (4) is

$$\Delta M \sim M ,$$

*i.e.*, the contribution to the mass from the modification of gravity term is of the same order as the mass itself!

In the following sections we will make concrete the statements pointed out above in the DGP model of modified gravity. They include the perturbative arguments leading to the derivation of the scale  $r_*$  and details on the exact solutions available for Schwarzschild-like sources and domain walls. While the former are based on an *ansatz*, legitimate concerns about the bulk boundary conditions of which were raised in [28], nevertheless, the fact that the *ansatz* recovers very precisely the 4D GR results at short distances and smoothly interpolates to the 5D regime (that no other solution is known to do), suggest that it may be capturing right physics. Moreover, the above properties were subsequently found to be true for the case of a domain wall for which an exact solution was obtained [29].

Although the existence of the mass deficit (4) could be interesting observationally, it may lead in certain cases to problems with the theory. Indeed, because of the terms (4) in the expression for a gravitational mass the proof of the positive energy theorem [30, 31] is not directly applicable. Hence, in general, there could exist negative “mass” solutions [5, 21] even for matter stress-tensors that satisfy conventional positive energy conditions. One example of this is the self-accelerated solution [5, 19]. Small perturbations about this solution in a linearized theory and with non-conformal sources exhibit ghost-like states [21, 23, 32, 33, 34], however, there exist serious arguments that the perturbative results cannot be trusted in the full non-linear theory [35] (see also [36]). Nevertheless, some semi-exact [27] and exact [29, 33, 37] non-perturbative solutions on the self-accelerated branch exhibit “negative mass”. This suggest that the self-accelerated branch should be unstable, however, it is not clear what is the time of its instability. An explicit calculation on decay of the selfaccelerated branch into the conventional one shows that such a decay does not take place, at lest in a quasi-classical approximation [38]. This question is still open and we will not be discussing it further in the present work.

If the mass screening is substantial, then, at scales beyond  $r_*$  gravity would be modified significantly. However, the value of the scale  $r_*$  for the entire observable Universe is  $H_0^{-1} \simeq 10^{28}$  cm. Therefore, on average, the beyond- $r_*$ -effects will be hard to detect. There may be exceptions for isolated clusters of galaxies separations between which are greater than their own  $r_*$  scales, and any other  $r_*$  scales in their vicinity<sup>4</sup>. For precise calculations of the beyond  $r_*$  physics, however, new averaging technique and non-perturbative calculational methods would be needed.

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<sup>4</sup>We thank Lam Hui and Roman Scoccimarro for discussions on these issues.

## 2 The $r_*$ -scale (Vainshtein scale)

In what follows we will concentrate on the concrete example provided by the DGP model [4] in which all interactions except gravity are confined to a 4D brane embedded in an infinite volume 5D empty space where gravity propagates.

The modification of gravity in this model is given in terms of the extrinsic curvature  $K_{\mu\nu}$  of the brane and reads:

$$\mathcal{K}_{\mu\nu} = m_c(K_{\mu\nu} - g_{\mu\nu}K) . \quad (5)$$

The 5D space has coordinates  $(x^\mu, y)$  with the brane at the surface  $y = 0$  and the 4D metric in (5) is  $g_{\mu\nu}(x^\mu, y = 0)$ .

The macroscopic distance  $r_*$  in the DGP model can be derived by considering the linearized analysis of the theory [4]. In particular we will focus on the one-graviton exchange amplitude between two sources whose gauge independent expression reads as follows:

$$\mathcal{A}_{1\text{-graviton}}(p, y) = \frac{T_{1/3}^2}{p^2 + m_c p} \exp(-p|y|) , \quad (6)$$

where

$$T_{1/3}^2 \equiv 8\pi G_N \left( T_{\mu\nu}^2 - \frac{1}{3} T \cdot T \right) , \quad (7)$$

and  $p^2$  is the square of the Euclidean brane 4-momentum. The pole at  $p^2 = 0$  has zero residue, and the second pole in (6) is on a non-physical Riemann sheet. The former implies the absence of a massless graviton in the exchange while the latter describes the propagation of a metastable state with lifetime  $\sim m_c^{-1}$ , which decays into a continuum of KK modes.

The striking feature of (7) is that in the  $m_c \rightarrow 0$  limit (in which the modification of gravity should disappear) the numerator does not reduce to the analogous expression in GR:

$$8\pi G_N \left( T_{\mu\nu}^2 - \frac{1}{2} T \cdot T \right) . \quad (8)$$

This fact, which could be used to exclude (7) by observations, is known as the van Dam-Veltman-Zakharov discontinuity (vDVZ) [39]. The difference between (7) and (8) is due to the fact that a 5D graviton (or a massive graviton for that matter) propagates 5 on-shell degrees of freedom (helicity-2, helicity-1, and helicity-0), while the GR graviton propagates only 2 on-shell degrees of freedom (helicity-2 state). And while the helicity-1 state of the 5D graviton does not contribute to (6) at the linearized level because of the contraction with conserved sources, the helicity-0 state couples to the trace of the energy-momentum tensor and gives a non-vanishing contribution (when  $T \neq 0$ ).

It has been argued in [18, 19] that the continuity in the  $m_c \rightarrow 0$  limit would be restored if nonlinear effects were taken into account. The relevance of these terms can be understood in the following terms: the longitudinal part of the graviton propagator in DGP contains terms proportional to  $p_\mu p_\nu / m_c p$ . This term does not contribute to the amplitude (6) because of conservation of the stress-tensor, but it does contribute already in the first nonlinear correction (since the stress-tensor is only covariantly conserved in the non-linear theory). And due to the singular behavior of these terms in the  $m_c \rightarrow 0$  limit, perturbation theory breaks down prematurely. However, this breakdown is an artifact of an ill-defined perturbative expansion – the known exact solutions of the model have no trace of breaking [19]. The perturbative expansion in powers of  $G_N$  gets “contaminated” by another dimensionful parameter  $1/m_c$ , and this leads to its breakdown. As possible ways forward one could either adopt a different type of expansion, e.g., an expansion in the small parameter  $m_c$  [19, 24], or try to find exact solutions<sup>5</sup>. Both of these programs have been carried out to a certain extent and we will review in the next section the salient features of the latter.

As presented in (2), the DGP model has one adjustable parameter, namely  $m_c$  which determines a scale that separates two different regimes of the theory. For distances much smaller than  $m_c^{-1}$  one would expect the solutions to be well approximated by GR and the modifications to appear at larger distances. This is indeed the case for distributions of matter and radiation which are homogeneous and isotropic at scales  $\gtrsim r_c$  [5, 6, 19]. However, more compact sources exhibit different properties. For example, a compact static source of the mass  $M$  and radius  $r_0$ , such that  $r_M < r_0 \ll r_c$  ( $r_M \equiv 2G_N M$  is the Schwarzschild radius) a new scale, combination of  $r_c$  and  $r_M$ , emerges (the so-called Vainshtein scale<sup>6</sup>) [19]:

$$r_* \equiv (r_M r_c^2)^{1/3}. \quad (9)$$

Below this scale the predictions of the theory are in good agreement with the GR results and above it they deviate considerably. These type of sources will be discussed in more detail in the next section together with other ones with higher simpler symmetry: domain walls.

### 3 Concrete examples

In this section we will focus on the mass screening phenomenon describing how it arises in the cases of the Schwarzschild-like non-perturbative solution (NPS) of [27, 43] and the exact domain wall (DW) solutions of [29].

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<sup>5</sup>It is also possible to modify the theory at the linearized level so that the conventional perturbative expansion is well-behaved [45, 46],[47],[48].

<sup>6</sup>A similar, but not exactly the same scale was discovered by Vainshtein in massive gravity [18], hence the name.

### 3.1 Schwarzschild solution

The NPS solution studied in [27, 43] is found by considering a static metric with spherical symmetry on the brane and with  $\mathbf{Z}_2$  symmetric line element:

$$ds^2 = -e^{-\lambda} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2 + 2\gamma dr dy + e^{\sigma} dy^2 , \quad (10)$$

where  $\lambda$ ,  $\gamma$ ,  $\sigma$  are functions of the radial coordinate on the brane  $r$  and the transverse direction coordinate  $y$ . The  $\mathbf{Z}_2$  symmetry across the brane ( $y = 0$ ) implies that  $\gamma$  is an odd function of  $y$  while the rest are even. The choice  $-g_{tt} = 1/g_{rr}$  represents an *ansatz*, but notice that we have kept the off-diagonal term  $g_{ry} = \gamma$ .

The brane is chosen to be straight in the above coordinate system but one could transform (10) to another one in which the metric is diagonal  $ds^2 = -A(r, z)dt^2 + B(r, z)d\rho^2 + C(r, z)d\Omega^2 + dz^2$ . Here,  $A \neq 1/B$  in general, and the *ansatz* is reflected in the fact that in this system our brane will be bent or, in other words, in a particular nontrivial choice of the position of the brane  $z(r)$ .

This *ansatz* allows us to close the system of equations *on the brane*, and leads to the following solutions for the gravitational potential  $\phi$ ,

$$e^{-\lambda} = 1 + 2\phi , \quad (11)$$

$$\phi(r) = \frac{3m_c^2}{4r} \int dr r^2 U(r) . \quad (12)$$

The function  $U(r)$  in (12) is given implicitly by the solutions of the following two equations (giving rise to a conventional and self-accelerated branch respectively):

$$(k_1 r)^8 = - \frac{(1 + 3U + f)}{U^2(3 + 3U + \sqrt{3}f)^{2\sqrt{3}}(-5 - 3U + f)} , \quad (13)$$

$$(k_2 r)^8 = - \frac{(-5 - 3U + f)(-3 - 3U - \sqrt{3}f)^{2\sqrt{3}}}{(U + 2)^2(1 + 3U + f)} , \quad (14)$$

where  $f = \sqrt{1 + 6U + 3U^2}$  and  $k$  is an integration constant.

The off-diagonal and  $yy$  metric components are determined from

$$\frac{4r^2(r\phi)_r}{(r\phi)_{rr}} = \frac{(r^4\gamma e^{-\lambda})_r}{(r\gamma e^{-\lambda})_r} , \quad (15)$$

$$e^{\sigma} = -m_c^2 \left[ \frac{(r^4\gamma e^{-\lambda})_r}{4r^2(r\phi)_r} \right]^2 + e^{-\lambda} \gamma^2 , \quad (16)$$

and the profile of the warp factors (“ $y$ -derivatives”) can be computed on the brane as well.

There are two integration constants,  $k$  and the one produced in the integration (12), which are determined by imposing appropriate boundary conditions near the source ( $r \ll r_*$ ) and at large distances. For the first condition we impose the 4D

behavior of the potential near the source:  $\phi = G_N M/r$ , while for the second one we require that the coefficient of the possible  $1/r$  term be zero, *i.e.*, to be left with 5D behavior at large distances, namely,  $\lambda \sim \tilde{r}_M^2/r^2$  in the conventional branch and  $\lambda \sim m_c^2 r^2 + \tilde{r}_M^2/r^2$  in the self-accelerated branch.

### 3.1.1 Conventional branch

The conventional branch is obtained from the solution of (13). As shown in [27, 43] the boundary conditions discussed above determine the asymptotic behavior of the solution. At short distances,  $r \ll r_*$  ( $U \rightarrow +\infty$ ), we get

$$\phi = -\frac{G_N M}{r} + \frac{1}{2}\alpha_1 m_c^2 r^2 \left(\frac{r_*}{r}\right)^{2(\sqrt{3}-1)} + \dots, \quad (17)$$

where  $\alpha_1 \approx 0.84$  and the coefficient of the  $1/r$  term was chosen to be  $-G_N M$  by fixing the constant of integration in (12).

The other integration constant  $k_1$  is chosen such that at large distances,  $r \gg r_*$  ( $U \rightarrow 0^+$ ), we obtain an expansion with no  $1/r$  term:

$$\phi = -\frac{\tilde{r}_{M_1}^2}{2r^2} + \dots, \quad (18)$$

which fixes the value of  $k_1$  in terms of  $r_*$  ( $(r_* k_1)^3 \approx 0.21$ ) and also implies

$$\tilde{r}_{M_1}^2 \approx 0.56 r_M r_*. \quad (19)$$

The relation (19) should be contrasted with the naive expectation from a linearized analysis: in the 5D regime one would have expected to have  $\tilde{r}_{M_1} \sim r_M r_c$ , however, we get a much smaller value, reduced by a factor  $r_*/r_c \equiv (r_M/r_c)^{1/3}$ .

Therefore, as we see, a short distance observer at  $r_M \ll r \ll r_*$  would measure the gravitational mass  $M$  with a small corrections to Newton's potential, while the large distance observer at  $r \gg r_*$  would measure an effective gravitational mass  $\sim M(r_M/r_c)^{1/3}$  [27]. The latter includes the effects of the 4D curvature which is significant up to  $r_*$  as depicted in figure 2. The 5D mass is partially screened at large distances.

Another point worth emphasizing is that a perturbative expansion suggests that for  $r_* \ll r \ll r_c$  the metric should have an approximately four-dimensional,  $1/r$ , scalar-tensor-gravity type form [4, 19]. However, the NPS above exhibits a different behavior: beyond  $r_*$  the metric turns into the one produced by a five dimensional source. We interpret this as a complete screening of the 4D mass of the source by the halo of non-zero curvature.

The screening of the 4D mass can be made explicit by taking into account the expression for the 4D Komar mass as a function of  $r$ . In the ansatz used here gives this gives an effective mass

$$M_{eff} \sim M_{Pl}^2 (-r\phi + r(r\phi)_r). \quad (20)$$



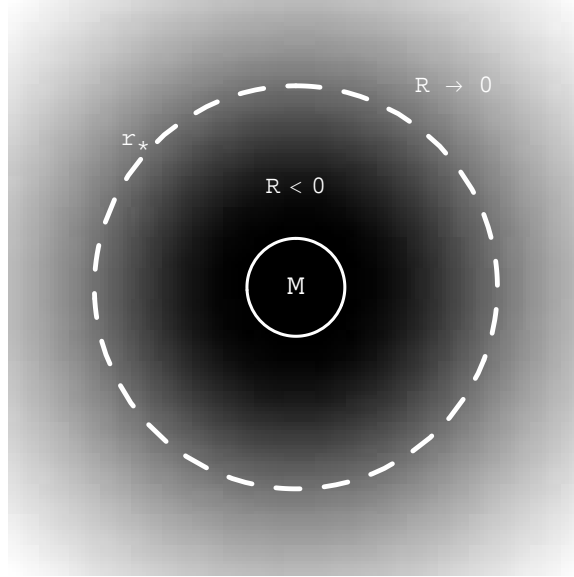


Figure 2: Conventional branch

The first term the rhs of (20) is a smooth decreasing function of  $r$  and gives a contribution that is  $\sim M$  (the original mass of the source) up to  $r_*$  and rapidly falls off like  $1/r$  beyond that point. The second term, on the other hand, depends on the gradient of  $\phi$  and is peaked around  $r_*$ . The combined effect is seen in figure 3: the effective mass increases from its 4D value  $M$  near the source up to  $r \sim r_*$  and then falls to zero abruptly.

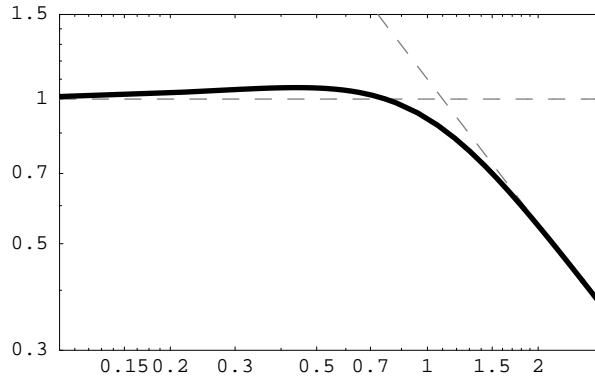


Figure 3: Log-Log plot of  $M_{eff}/M$  vs.  $r/r_*$ . The oblique dashed line shows a  $1/r$  fall-off.

The short distance mass increase can also be deduced from the approximate form of the potential (17) since the second term provides an additional attraction toward the source.

### 3.1.2 Self-accelerated branch

The solution on the self-accelerated branch is obtained from (14). The relation between  $k_1$  and  $r_*$  is obtained, as in the conventional case, by imposing boundary conditions together with the following asymptotic behavior. At large distances,  $r \gg r_*$  ( $U \rightarrow -2^-$ ), we derive

$$\phi = \frac{\tilde{r}_{M_2}^2}{2r^2} - \frac{1}{2}m_c^2 r^2 + \dots, \quad (21)$$

where,

$$\tilde{r}_{M_2}^2 \approx 0.45 \, r_M r_* , \quad (22)$$

, *i.e.*, 5D mass screening. At short distances,  $r \ll r_*$  ( $U \rightarrow -\infty$ ), we get

$$\phi = -\frac{G_N M}{r} + \frac{1}{2}\alpha_2 m_c^2 r^2 \left(\frac{r_*}{r}\right)^{2(\sqrt{3}-1)} + \dots, \quad (23)$$

where  $\alpha_2 = -\alpha_1 \approx -0.84$  is, in absolute value, the same constant appearing in the conventional branch short distance expansion (17). Note, however, that the sign of the correction to the 4D behavior is opposite in the two branches.

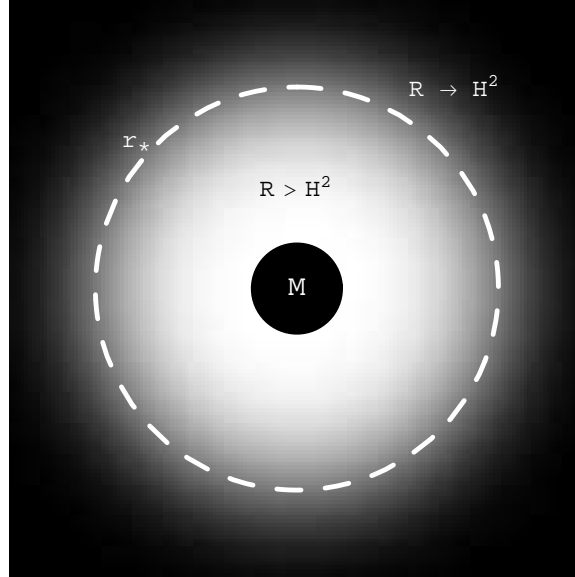


Figure 4: Self-accelerated branch

At intermediate distances,  $r_* \ll r \ll r_c$ , the potential contains a 5D gravitational term that is *repulsive*,  $\tilde{r}_M^2/r^2$ . This looks like a 5D negative mass. However, this is not an asymptotic value of the mass since one can only cover the solution in the above coordinate system till  $r \sim r_c$  where the dS like horizon is encountered.

Moreover, in the intermediate regime  $r_* \ll r \ll r_c$ , the de Sitter term  $m_c^2 r^2$  in the potential always dominates over the  $\tilde{r}_M^2/r^2$  term suggesting that the effects due to the Schwarzschild source are strongly suppressed. The picture that explains screening in this branch is depicted in figure 4.

### 3.2 Domain Walls

The second example that illustrates the screening phenomenon in the DGP model is that of a brane DW source. The study of this type of sources was done in [29] on which we base the following discussion.

The source considered is a Nambu-Goto DW in 4D localized at  $z = 0$  with stress-tensor

$$T_{\mu\nu} = \sigma \delta(z) \text{diag}(1, -1, -1, 0) , \quad (24)$$

where  $\sigma$  is the tension of the wall and  $z$  denotes the coordinate transverse to its world-volume spanned by  $(t, x, y)$ .

The domain wall solution for such a source in GR [49, 50] displays 3D de Sitter expansion in its world-volume at a rate  $H = 2\pi G_N \sigma$ .

In DGP, however, the situation is different. For tensions smaller than a critical value,  $\sigma_c \equiv M_*^3 = m_c M_{Pl}/2$ , the wall has no gravitational effects. One way to understand this is by noticing that for these sources, the modification of gravity term (5) precisely compensates the energy momentum tensor  $T_{\mu\nu}$ . Therefore, the tension of the wall, as seen from the point of view of a 4D observer, is screened entirely by gravitational effects encoded in the extrinsic curvature. Not surprisingly, the domain wall world-volume remains flat, and so does the metric on the brane.

Furthermore, this screening takes place inside the core of the wall. Hence, the analog notion of the  $r_*$  scale for a domain wall (understood as where the self-shielding takes place) coincides with its thickness,

$$r_*^{(DW)} = d_{core} .$$

This is to be compared to the Schwarzschild-like case, where the shielding also occurs, and  $r_*$  extends outside the source. The net result is the screening of the 4D tension/mass in both cases.

For supercritical tension branes the extrinsic curvature can no longer balance the energy momentum tensor and the brane inflates. The transverse direction to the wall is compactified to a size  $d$  and a zero mode graviton appears. Since this phenomenon also takes place in GR, it provides a means to contrast the 5D effects in DGP with those of a supercritical DW in 5D, *i.e.*, by comparing the world-volume inflation rate in both cases.

The exact solutions of [29] give a suppression factor for the inflation rate of the supercritical DW in DGP for both branches of solutions. In the conventional branch it is  $d/2r_c$  while in the self-accelerated branch it is given by  $-d/2r_c$ . As

argued above, we should identify the Vainshtein scale  $r_*$  for these sources  $d$  and therefore we find the same parametric screening of the 5D tension as for the mass in the Schwarzschild-like solutions of the previous example.

## 4 Discussions

The above results may also be applicable to other models where the results of 4D gravity are recovered through strongly coupled behavior. The minimal model of brane induced gravity in greater than five dimensions [51] contains ghosts [52]. However, its extensions are ghost free [53], a small subset of which has a strong coupling regime [53, 54] (see, also [55]). A recent model of cascading brane induced gravity is also ghost free [56]. It would be interesting to understand the issue of presence/absence of the mass screening in these models (see, also, [57]).

Unfortunately, at present there is no consistent 4D theory of Lorentz invariant massive gravity, as it suffers from nonlinear instabilities [58, 59, 60, 61]. However, the mass screening effect described above is based on rather universal principles, and it is reasonable to expect that the phenomenon will also persist if a consistent model is found.

It would be interesting to understand the issue of the mass screening in the  $f(R)$ -type models of modified gravity, see, e.g., [62], as well as in Lorentz violating models [63, 64, 65, 66, 67]. Some of these models [64, 65, 66, 67] avoid the strong coupling behavior, in which case the mass screening is not expected to be very significant.

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